

A Theory of Quantum Observation and the Emergence of the Born Rule

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The universe we observe requires a twofold concept of locality. On one hand there are the strictly Einstein-local interactions that generate the time evolution, on the other hand the quantum state space requires a non-local description of multiple particle correlations.

This article demonstrates that an observer in such a universe has to rely on local interactions to learn about his environment. He is therefore severely restricted in his ability to reconstruct the local physical universe. We argue that this reconstruction is the defining process of observation. The reconstructed quantum dynamics are shown to be non-unitary and non-linear in general, even if the system evolves unitarily on a global scale.

Interactions with massless free particles are found to have great influence on observation. The special case of a scattering process with an uncharged massless vector boson can result in a stochastic process conforming to the Born rule. Based on this result, a theory of quantum measurement, that describes a measurement device as a cascade of certain scattering events is formulated.

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I. INTRODUCTION

Quantum theory is an enormously successful theory. We can use it to predict the behavior of nature up to the limits of what is measurable. This is even more astounding given the fact that we have almost no understanding of what measuring really means. Quantum theory equips us with an algorithm that describes how the possible outcomes of a measurement can be calculated and predicted statistically. The measurement postulate of quantum theory does not define what a measurement instrument physically is, nor how it could possibly function. Even worse, it appears that the process of measurement is not compatible with the smooth unitary evolution which is fundamental to quantum theory.

A lot of effort has been placed into resolving this measurement problem and has resulted in a collection of interpretations of quantum theory [4, 7]. Despite significant progress in some areas like decoherence theory, a generally accepted solution of the measurement problem is not currently known. One of the most important ingredients of such a solution has to be a derivation of the statistical law, known as the Born rule [1], of quantum observations from fundamental properties of bare quantum theory without the measurement postulate. While such derivations have been attempted [2, 5, 8], they usually involve non-obvious assumptions or have been shown to use circular arguments.

This article attempts to derive the discontinuous and random yet stable results of observation as stated in the

measurement postulate only from generally accepted first principles. This is done by assuming the position of an observer that collects information about his environment. It is this approach that qualifies the results as a true theory of quantum measurement.

Everything we know about the universe is a result of our interaction with it while being a part of it. As obvious as this statement may seem, it is of fundamental importance to our understanding of the way we describe the universe, create physical theories and interpret them. Even in the non-local setting of quantum theory all interactions are subject to locality. And if the universe is relativistic, an observer is limited to his light cone and does not have access, in the form of interaction, to the complete state of the universe.

The information available to the observer and the change of knowledge during measurement is also part of the Copenhagen Interpretation of quantum theory, or more precisely, the subjectivism represented by Werner Heisenberg [3]. However, his idea has not evolved into a quantitative theory and it does not specify the exact nature of the lack of knowledge. Obtaining a quantitative description of the information available to the local observer corresponds to describing the state of the universe that the observer would be able to reconstruct just from interaction without any additional assumptions.

Consider a simplified relativistic universe containing just two particles and an observer located close to one of these particles. The particles are supposed to have interacted a long time ago and are spatially separated to make sure that they cannot interact for another sufficiently long time. The observer, only being able to interact with one of the particles, is not in the position to tell if the two particles are entangled. In fact, he is

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unable to tell if a second particle exists at all. Making no unfounded assumptions implies that his description of the universe must not contain the other particle. The state he reconstructs is specifically not a mixed state, because that too would require knowledge about the existence of the other particle. He is left with the option of describing a single particle with a pure state, as we will discuss later.

A different, but similar scenario is created by the interaction of the local system with free massless particles. Instead of a distant particle, imagine a photon that has just been emitted into the environment. The information it carries is inaccessible as soon as it leaves the observer's horizon. Incoming photons also only become accessible the very instant they reach the observer. The moment of interaction then creates a discontinuity in the observer's best guess for the state of the universe. The transport of information to and from the observer by interaction with the radiation field also creates a source randomness. The unknown state of the incoming radiation influences the local system in a way unpredictable to the local observer. The effect of the random perturbation is not entirely unpredictable however. It is restricted by the nature of the interaction and shaped by the way the system couples to the radiation field. This opens the door for a statistical distribution that correlates with the previously known state of the system.

II. THE POSTULATES

In order to build the theory of local observation onto firm grounds, we specify the following postulates. They formalize the assumptions used in the derivations presented.

1. The evolution of the state of the universe is unitary and generated only by interactions within the universe.
2. The interactions in the universe are local and in agreement with Special Relativity.
3. The observer is part of the universe and physically realized within a finite region of spacetime.
4. The description of the universe reconstructed by the observer is based on his local state history without any additional assumptions.

These postulates also describe the steps taken to arrive at a subjective description of the universe, starting from a hypothetical objective description. As observers,

we do not have access to the state of the universe, so this reduction of information from top to bottom might seem unnatural. We have to rely on Occam's principle to argue that a theory that predicts local observations from an unknown global mechanisms is valid if the assumptions made on the global scale are very plain and simple. This is true for the postulates listed above and therefore supports the postulate of a global unitary evolution.

Postulate 4 not only restricts the description of the universe to the information processing happening in the observer system. It also defines what we can regard to be physically distinguishable properties of the universe. Properties that no imaginable observer can distinguish cannot be part of the physical description of the universe. In other words, it is only the dynamical sequence of the states of the universe that results in an emergent physical reality.

In addition to these postulates we have to make one additional assumption about the universe. That is the existence of a free massless neutral vector boson field that interacts with the observer. The photon takes this role in our universe.

III. LOCAL STATE RECONSTRUCTION

A. The state of the universe

Our first postulate demands unitary evolution of the universe, but does not specify the state space. Instead, we will try to construct the state space from first principles.

The state space must be able to hold unitary representations of the observed symmetries of the universe. It must also be able to encode superposition and interference of states. All requirements are fulfilled by a finite dimensional complex vector space with a sesquilinear inner product. The inner product allows unitary transforms to be defined. It is not entirely unreasonable to assume that the state space is finite dimensional¹, but it is mathematically more convenient to allow for infinite dimensions. In this case the convergence of the inner product must be listed as an additional requirement, and we arrive at the well known separable Hilbert space \mathcal{H} of square integrable functions.

¹ The representation theory of the Lorentz group $SO(3,1)$ shows that a unitary representation requires an infinite dimensional space. So a finite dimensional quantum theory implies that the Lorentz group can only be realized approximately.

Another possible state space that meets all requirements is the space of certain linear operators on the specified Hilbert space. We can embed the Hilbert space vectors as projectors within this larger space and inherit the inner product in the form of the trace of an operator product. Again, for an infinite dimensional state space this trace must exist, so one is limited to trace class linear operators.

The dynamic evolution of the universe acts on these state spaces with a time dependent unitary transform. The transform is one sided for the Hilbert space and two sided for the operator space. This is also true for all other unitary symmetry transforms.

$$|\phi(t)\rangle = U(t_0, t) |\phi(t_0)\rangle \quad (1)$$

$$\Phi(t) = U(t_0, t)\Phi(t_0)U(t_0, t)^\dagger \quad (2)$$

Following postulate 4 only the sequence of states generated by the dynamical evolution is of physical relevance. As a result, state descriptions that are equivalent by a bijection f that commutes with the evolution are physically indistinguishable, because it allows switching back and forth between two representations at any point in time.

An important bijection is the left multiplication of $\Phi(t)$ with a unitary, possibly time dependent but predictable, operator that commutes with the evolution of the universe. This operation can be undone at any point in time and satisfies:

$$U(t_0, t)f(\Phi)U^\dagger(t_0, t) = f(U(t_0, t)\Phi U^\dagger(t_0, t)) \quad (3)$$

So $f(\Phi)$ is a redundant representation of Φ . However this redundancy does not correspond to an actual symmetry of the universe, because those symmetries act on both sides at the same time. Therefore, these states are not significant for the description of the universe and have to be removed in a unique state description. The subspace of *Hermitian* trace class operators does not allow this one sided transform and strips the redundant states from the state space. The *non-negative* Hermitian trace class operators \mathcal{P} are even exactly the quotient of the trace class operators and equivalence by left (or right) unitary multiplication.

Both possible state spaces \mathcal{H} and \mathcal{P} can be reduced even further. The linear structure of the spaces allows for scalar multiplication as bijection that trivially commutes with the dynamics. The two equivalence relations

$$(|a\rangle, |b\rangle) \in R_1 \Leftrightarrow \exists c \neq 0 \in \mathbb{C} : |a\rangle = c|b\rangle \quad (4)$$

$$(A, B) \in R_2 \Leftrightarrow \exists r > 0 : A = rB \quad (5)$$

generate the quotient spaces \mathcal{H}/R_1 and \mathcal{P}/R_2 of quantum states in the Hilbert space \mathcal{H} and the non-negative trace class Hermitian operator space \mathcal{P} respectively.

The state space \mathcal{P}/R_2 is usually constructed as the space of “classical ensembles” of quantum systems realized by the convex sum of projectors on the Hilbert space \mathcal{H} . It is also used to describe the state of subsystems by tracing over the environment. The first of these two uses relies strongly on the prior existence of the measurement postulate and cannot be used here. As for the second, tracing over the environment is not a possible operation when describing the whole universe. The state space \mathcal{P}/R_2 was specifically constructed from basic requirements to show that it is a valid state space just like the projective Hilbert space \mathcal{H}/R_1 , with no need for additional interpretation as space of ensembles or subsystem states. This is an important insight for the arguments further down.

Postulate 4 demands states that are dynamically indistinguishable result in the same physics and should be regarded as one single state. Consider a state $\Phi(t_0) \in \mathcal{P}$ and its evolution:

$$\Phi(t) = U(t_0, t)\Phi(t_0)U(t_0, t)^\dagger \quad (6)$$

Then with the unitarity of U it follows that

$$\Phi(t)^2 = U(t_0, t)\Phi(t_0)U(t_0, t)^\dagger U(t_0, t)\Phi(t_0)U(t_0, t)^\dagger \quad (7)$$

$$= U(t_0, t)\Phi(t_0)^2 U(t_0, t)^\dagger \quad (8)$$

meaning that Φ^2 evolves in exactly the same way as Φ and at any point in time we can switch back and forth these two states without making a difference dynamically, because squaring is a bijection on non-negative Hermitian operators and commutes with the evolution. The two states Φ and Φ^2 are dynamically equivalent and as such both describe the same physical system. The same is true for all natural powers of a state Φ , as they are all bijections. More generally, let

$$g : \mathbb{R}_{\geq 0} \longrightarrow \mathbb{R}_{\geq 0} \quad (9)$$

be an analytic bijection. Such a function is then necessarily strictly monotonic and $g(0) = 0$. It can be analytically extended to non-negative Hermitian operators and is a bijection there too. Any such g creates a state that is dynamically equivalent to the state it acts on. The equivalence classes generated by these bijections g are the *true* physical states.

One set of equivalence classes can be directly identified: The projectors in \mathcal{P} are invariant under g , up to a scalar multiplication, and each forms an equivalence class of its own. When g acts on a state Φ it preserves two properties: The mutually orthogonal eigensubspaces are invariant and the ordering of the eigenvalues is also strictly invariant because g is strictly increasing on the non-negative real numbers. The number of

eigenvalues is countable because \mathcal{H} is separable, and the trace-class bounds the sum of the non-negative eigenvalues, so that the maximum of the eigenvalues exists. It follows that a possible representation of the equivalence classes of physical states then consists of a *list of orthogonal eigensubspaces* in order of decreasing eigenvalues, and nothing else. Particularly, the eigenvalues do not have to be specified, as they change with g . For a finite list the last entry is the nullspace, which is invariant under g . It is redundant because it is the unique orthogonal complement of the direct sum of the other list entries. Therefore, this last entry can be omitted.

Depending on the equivalence class, this list can have a different number of entries. For a simple projector it only has one entry, the eigensubspace of the maximum eigenvalue which trivially equals 1. That is also the one property that all equivalence classes share. All equivalence classes can be partially characterized by the eigensubspace of the largest eigenvalue, which *always* exists.

Applying the unitary evolution of the universe to the eigensubspace lists shows that the evolution acts independently on each subspace and does not change the dimensions of the subspaces or the length of the list. The character of the equivalence classes is therefore invariant under time evolution. This effectively splits up the state space into an infinite set of state spaces with common character and each connected by unitary transforms.

So with all these considerations in place, the result is an infinite number of possible state spaces that allow for unitary evolution of the universe and do not encode symmetries that are indistinguishable by the evolution dynamics. Each state space is fully characterized by a finite or infinite sequence of natural numbers, including countable infinity, describing the dimension of a subspace. The evolution acts unitarily on each subspace. Furthermore, the original Hilbert space \mathcal{H} is contained in the set of state spaces as the sequence (1), representing a single one dimensional subspace that the dynamic acts on.

The n -dimensional projectors state space (n) creates a physical universe that slightly differs from \mathcal{H} . One can decide to describe such a universe with a single vector from the eigensubspace and will get consistent dynamics. So the one dimensional state description is not unique, and even several one dimensional descriptions could be used at the same time. This universe can be understood as a generated by n non-interfering superpositions of orthogonal, but not uniquely determined \mathcal{H} universes. Observers would notice the absence of interference in certain situations. But for finite n that effect would be so rare that such a universe is practically indis-

tinguishable from a (1) universe. Furthermore a coincidence of eigenvalues to form a more than 1-dimensional eigensubspace is very unlikely, in fact those configurations form a subset of measure zero for all practical measures.

For universes with a state space with more than one relevant subspace (n, \dots) the dynamics is dominated by the first listed subspace. This can be seen by applying an explicit bijection

$$g_k(\Phi) = \Phi^k \quad (10)$$

for natural k to a non-negative trace class Hermitian operator. For sufficiently large k the largest eigenvalue dominates as strongly as desired. So the dynamics of the equivalence class representatives realized as a density matrix can get arbitrarily close to the dynamics of the states in (n) and for $k \rightarrow \infty$ we recover that space as the limit. For this reason there is no new physics to be expected in (n, \dots) universes compared to an (n) universe. It follows that the map g_k for $k \rightarrow \infty$, corresponding to the projection onto the eigensubspace with the largest eigenvalue, is a useful tool for reducing the complexity of the dynamical description in non-projection state space universes.

Summing up, there is little to no reason, not to assume that the universe is a (1) (i.e. traditional \mathcal{H}) universe. At this point the rather involved discussion of possible state spaces and their physical implications may not seem to be very useful. The discussion will prove to be of importance later on however. The same arguments used for the complete universe also apply to the local observer, but with the additional constraint of limited information. So the classification described here is the guide to how an observer will reconstruct the state of the universe from his interactions.

B. Local interactions and available state information

One of the most obvious weaknesses of the measurement postulate of traditional quantum theory is the lack of a precise definition of an observer. For our purpose an observer must be a mechanism equipped with a memory and realized by physical interactions in the universe. It analyzes its environment, stores information and makes predictions based on these observations. This comes with the implicit emergence of a certain subjective reality for the observer. If we assume that such an observer has a limited observation time span it also follows that the essential parts of the mechanism are constrained to a bounded region of space due to the

limited propagation speed of the interactions they are based upon.

An observer of this kind can be human, ideally a physicist, but it does not have to be. The complexity and details of the mechanism is not of importance for the essential properties of such an observer. Accordingly, these aspects will not be part of the model. The mere presence of an observer within the system will be assumed, even if the number of degrees of freedom is much too small to contain any sensible realization of such an observer mechanism². It will also be assumed that the observer is massive and that the quantum system of interest is described as seen from the observer's rest frame.

The information the observer gathers about the rest of the universe is a result of his interacting with the environment. With the interaction propagation speed limit in place, the knowledge gained within the observation time is clearly limited. The exact description of this limitation is the subject of this chapter.

1. Stripping the universe

In its rest frame we can assume the relevant parts of the observer to be contained in a spherical spatial region of radius r_l . The universe outside of this radius is not part of the observer mechanism, but it does influence the observer by interaction. This influence only reaches up to a radius

$$r_h := r_l + cT \quad (11)$$

limited by the observation time T of the observer and the speed limit c for the propagation of interactions. This radius is a strict horizon for the information the observer can recover about the universe he lives in. Anything outside the horizon is dynamically inaccessible and therefore not part of his experience of reality.

In a *classical* universe we could simply remove all objects or fields on the far side of the horizon and the observer would not notice. On the other hand, the observer can only reconstruct the state of the universe as one without any structure outside the horizon if he does not want to rely on arbitrary guesswork.

In principle the same applies to a *quantum* universe, but with a non-trivial twist. Each particle carries its own copy of spatial information, instead of sharing one spatial background like in a classical universe. Starting with a single particle Hilbert space \mathcal{H}_1 and ignoring indistinguishability, we can define the n -particle space by taking the tensor product power

$$\mathcal{H}_n := \mathcal{H}_1^{\otimes n} \quad (12)$$

and in the next step the Fock space of the particles as the direct sum of all possible n -particle spaces. The one dimensional vacuum space is denoted by $\mathcal{H}_1^{\otimes 0}$.

$$\mathcal{H}_F := \bigoplus_{n=0}^{\infty} \mathcal{H}_1^{\otimes n} \quad (13)$$

For a single particle space we can introduce two projection operators P_a and P_i that project onto the dynamically accessible and inaccessible states respectively. Naturally $P_a + P_i = \mathbf{1}$ must hold. In case that accessibility is determined by the horizon with radius r_h as defined above, we can explicitly express the projectors in the position basis of the single particle Hilbert space.

$$P_a := \int_{r \leq r_h} |\mathbf{r}\rangle \langle \mathbf{r}| d^3\mathbf{r} \quad (14)$$

$$P_i := \int_{r > r_h} |\mathbf{r}\rangle \langle \mathbf{r}| d^3\mathbf{r} \quad (15)$$

The eigensubspaces of both of these projectors, \mathcal{H}_a and \mathcal{H}_i , hold the accessible and inaccessible states. We can write the single particle Hilbert space as a direct sum of these two orthogonal subspaces.

$$\mathcal{H}_1 = \mathcal{H}_a \oplus \mathcal{H}_i \quad (16)$$

Expanding the Fock space definition with this substitution results in

$$\begin{aligned} \mathcal{H}_F = & \mathcal{H}_1^{\otimes 0} \oplus \bigoplus_{n=1}^{\infty} \mathcal{H}_a^{\otimes n} \oplus \bigoplus_{n=1}^{\infty} \mathcal{H}_i^{\otimes n} \\ & \oplus \bigoplus_{n,m=1}^{\infty} S(\mathcal{H}_a^{\otimes n}, \mathcal{H}_i^{\otimes m}) \end{aligned} \quad (17)$$

where $S(\mathcal{H}_a^{\otimes n}, \mathcal{H}_i^{\otimes m})$ is the symmetrized direct sum of the tensor product of n times \mathcal{H}_a and m times \mathcal{H}_i . For example the symmetrization $S(\mathcal{H}_a^{\otimes 2}, \mathcal{H}_i^{\otimes 1})$ expands to:

$$\begin{aligned} S(\mathcal{H}_a^{\otimes 2}, \mathcal{H}_i^{\otimes 1}) = & (\mathcal{H}_a \otimes \mathcal{H}_a \otimes \mathcal{H}_i) \\ & \oplus (\mathcal{H}_a \otimes \mathcal{H}_i \otimes \mathcal{H}_a) \\ & \oplus (\mathcal{H}_i \otimes \mathcal{H}_a \otimes \mathcal{H}_a) \end{aligned} \quad (18)$$

² This is not really a constraint on the theory. We will simply be using the same information gathering methods that are available to a hypothetical observer without being interested in its actual physical details. As we will see, this approach is entirely sufficient.

If we want to strip the inaccessible information from \mathcal{H}_F in equation (17) we can first drop³ the pure \mathcal{H}_i powers, similarly to the classical situation. The remaining inaccessible parts of the state space appear in tensor products with the accessible subspace. Removing these inaccessible parts while keeping as much as possible of the accessible parts intact will also remove relative phase information. The resulting states will add incoherently and cannot be represented in the same Fock space \mathcal{H}_F .

For the stripped state space we must rely on the trace class non-negative hermitian operators on \mathcal{H}_F denoted by $T(\mathcal{H}_F)$, with the canonical embedding of the first as projections in the latter. The dynamics preserving map

$$S(\mathcal{H}_a^{\otimes n}, \mathcal{H}_i^{\otimes m}) \rightarrow T(\mathcal{H}_a^{\otimes n}) \quad (19)$$

can then be realized by tracing over the inaccessible tensor factor spaces.

Let $|\Psi\rangle$ be a Fock state and $|\psi_n\rangle$ its n -particle component.

$$|\Psi\rangle = \sum_{n=0}^{\infty} c_n |\psi_n\rangle \quad (20)$$

The projection operators $P_a^{(k,n)}$ and $P_i^{(k,n)}$ act on the k -th particle in the n -particle subspace. Stripping only the 1-particle subspace starts with distributing the projectors over the state.

$$\begin{aligned} |\psi_1\rangle &= \left(P_a^{(1,1)} + P_i^{(1,1)} \right) |\psi_1\rangle \\ &= P_a^{(1,1)} |\psi_1\rangle + P_i^{(1,1)} |\psi_1\rangle \end{aligned} \quad (21)$$

If the particle is entirely accessible and therefore

$$P_i^{(1,1)} |\psi_1\rangle = 0 \quad (22)$$

we already have a stripped state. If the particle is entirely inaccessible we have

$$P_a^{(1,1)} |\psi_1\rangle = 0 \quad (23)$$

and dropping the inaccessible part would result in a physically undefined state, the 0-vector. Instead, the observer should simply find no particle, or in other words, the vacuum state $|\psi_0\rangle$. The vacuum state can already be part of $|\Psi\rangle$ however, and the observed subjective vacuum must not interfere with the true vacuum. The two vacua must add incoherently. To simplify the notation we can identify the vacuum space $\mathcal{H}_1^{\otimes 0}$ with the complex scalars and choose:

$$|\psi_0\rangle = 1 \in \mathbb{C} \quad (24)$$

So if we apply the stripping map

$$\Lambda : \mathcal{H}_F \rightarrow T(\mathcal{H}_F) \quad (25)$$

to the vacuum and single particle subspaces we get

$$\begin{aligned} \Lambda(c_0 |\psi_0\rangle + c_1 |\psi_1\rangle) & \\ &= \Lambda\left(c_0 |\psi_0\rangle + c_1 P_a^{(1,1)} |\psi_1\rangle + c_1 P_i^{(1,1)} |\psi_1\rangle\right) \\ &= T\left(c_0 |\psi_0\rangle + c_1 P_a^{(1,1)} |\psi_1\rangle\right) + \text{tr}_{\mathcal{H}_1}\left(T\left(c_1 P_i^{(1,1)} |\psi_1\rangle\right)\right) \end{aligned} \quad (26)$$

where

$$T(|\phi\rangle) := |\phi\rangle \langle \phi| \quad (27)$$

is the natural embedding. The trace results in a complex number representing the vacuum state.

If we also consider two-particle states we get additional terms from

$$\begin{aligned} |\psi_2\rangle &= \left(P_a^{(1,2)} + P_i^{(1,2)} \right) \left(P_a^{(2,2)} + P_i^{(2,2)} \right) |\psi_2\rangle \\ &= P_a^{(1,2)} P_a^{(2,2)} |\psi_2\rangle + P_i^{(1,2)} P_i^{(2,2)} |\psi_2\rangle \\ &\quad + P_a^{(1,2)} P_i^{(2,2)} |\psi_2\rangle + P_i^{(1,2)} P_a^{(2,2)} |\psi_2\rangle \end{aligned} \quad (28)$$

The first term is not changed by Λ , the second term is mapped to the vacuum and the last two terms are traced over to result in an incoherently mixing single particle state.

$$\begin{aligned} \Lambda(c_0 |\psi_0\rangle + c_1 |\psi_1\rangle + c_2 |\psi_2\rangle) & \\ &= T\left(c_0 |\psi_0\rangle + c_1 P_a^{(1,1)} |\psi_1\rangle + c_2 P_a^{(1,2)} P_a^{(2,2)} |\psi_2\rangle\right) \\ &\quad + \text{tr}_{\mathcal{H}_i}\left(T\left(c_1 P_i^{(1,1)} |\psi_1\rangle + c_2 P_a^{(1,2)} P_i^{(2,2)} |\psi_2\rangle\right.\right. \\ &\quad \left.\left.+ c_2 P_i^{(1,2)} P_a^{(2,2)} |\psi_2\rangle\right)\right) \\ &\quad + \text{tr}_{\mathcal{H}_i^{\otimes 2}}\left(T\left(c_2 P_i^{(1,2)} P_i^{(2,2)} |\psi_2\rangle\right)\right) \end{aligned} \quad (29)$$

The trace is over all terms with a single inaccessible particle. The different order of the traced tensor factor space is not problematic. Reordering the inaccessible spaces does not change the result as they are traced over eventually. So we can think of that reordering implicitly happening while tracing.

As can be seen, this becomes tedious and complicated to write down for an increasing number of particle number eigenspaces. The definition of Λ is a lot easier if we express it in terms of an occupation number basis of \mathcal{H}_F , starting from a single particle eigenbasis of the projection operators P_a and P_i and assuming indistinguishable particles this time. With the bases $|k\rangle_a$ and $|k\rangle_i$ for \mathcal{H}_a and \mathcal{H}_i respectively and natural k we can characterize a fully symmetric n -particle state by listing the number of particles $n_{k;a}$ and $n_{k;i}$ in each accessible

³ As will be discussed further on, they actually have to be replaced with a vacuum state.

and inaccessible single particle base state respectively. The total particle number is then

$$n = \sum_{k=0}^{\infty} (n_{k;a} + n_{k;i}) \quad (30)$$

and we can label the state

$$|(n_{0;a}, n_{1;a}, n_{2;a}, \dots); (n_{0;i}, n_{1;i}, n_{2;i}, \dots)\rangle \quad (31)$$

or in a more compact form

$$|\mathbf{n}_a; \mathbf{n}_i\rangle \quad (32)$$

with the occupation lists \mathbf{n}_a and \mathbf{n}_i respectively. For our convenience we define

$$|\mathbf{n}_a| := \sum_{k=0}^{\infty} n_{k;a} \quad (33)$$

and likewise for $|\mathbf{n}_i|$, so that:

$$n = |\mathbf{n}_a| + |\mathbf{n}_i| \quad (34)$$

With the occupation number basis of the Fock space it can be seen from direct calculation that $|\mathbf{n}_a; 0\rangle$ remains unchanged under Λ , while states with inaccessible single particle states are mapped to stripped states,

$$|\mathbf{n}_a; \mathbf{n}_i\rangle \mapsto T(|\mathbf{n}_a; 0\rangle) \quad (35)$$

which already includes the special case of all inaccessible states being mapped to the vacuum. The full stripping map is then defined for both bosonic and fermionic states and explicitly given by the following expression.

$$\begin{aligned} \Lambda : \mathcal{H}_F &\longrightarrow T(\mathcal{H}_F) \\ |\Psi\rangle &\longmapsto \Lambda(|\Psi\rangle) \\ \Lambda(|\Psi\rangle) &:= \sum_{\mathbf{n}_i} T\left(\sum_{\mathbf{n}_a} |\mathbf{n}_a; 0\rangle \langle \mathbf{n}_a; \mathbf{n}_i | \Psi \rangle\right) \end{aligned} \quad (36)$$

For an entirely accessible state we get

$$\Lambda(|\Psi\rangle) = |\Psi\rangle \langle \Psi| \quad (37)$$

while fully inaccessible states are mapped to the vacuum.

The construction of the stripping map guarantees that an observer interacting with the universe will not be able to distinguish the real state of the universe $|\Psi\rangle$ from the stripped state $\Lambda(|\Psi\rangle)$. Therefore the observer has no way of reconstructing the real state of the universe. But can he theoretically even reconstruct the stripped state? The answer must be that he cannot, for the following two reasons.

First, the classification of accessible and inaccessible states based on the horizon radius discussed above is not very precise. While all states marked as dynamically inaccessible are in fact certainly not accessible, not all states marked as accessible really are accessible. That means the stripped state contains information that is not available to the observer even under ideal conditions. We will discuss this in more depth below when we look at the process of observing an experiment.

The second reason is the inability of the observer to distinguish state representations that are equivalent regarding their unitary evolution. Like discussed in the previous section, $\Lambda(|\Psi\rangle)$ is for example intrinsically indistinguishable from its positive integer powers. Consequently the observer does not have enough information to reconstruct the stripped state without further assumptions about the state of the universe. The *only* assumption that does not require the addition of arbitrarily made up information is that the reconstructed state of the universe is a *pure* state. Following the earlier discussion about the state of the universe, we define the normalized stripped state in \mathcal{H}_F as the normalized limit of the bijection (10).

$$\begin{aligned} \bar{\Lambda} : \mathcal{H}'_F &\longrightarrow \mathcal{H}_F \\ |\Psi\rangle &\longmapsto T^{-1}(\bar{\Lambda}(|\Psi\rangle)) \\ \bar{\Lambda}(|\psi\rangle) &:= \lim_{k \rightarrow \infty} \frac{\Lambda(|\Psi\rangle)^k}{\text{tr}(\Lambda(|\Psi\rangle)^k)} \end{aligned} \quad (38)$$

The limit is a projection operator onto the eigensubspace of $\Lambda(|\Psi\rangle)$ with the greatest eigenvalue. For a pure state the eigensubspace must be one dimensional, which can then be naturally⁴ mapped back to the Hilbert space \mathcal{H}_F by T^{-1} . If we assume that the eigenvalues are essentially random then it is extremely unlikely that the two greatest eigenvalues are exactly equal. So it seems to be safe to assume that $\bar{\Lambda}(|\Psi\rangle)$ practically always results in a pure state. The states that do not get mapped back to a pure state are formally removed from \mathcal{H}'_F . The time evolution of the resulting state is not unitary and even non-linear.

2. Making an observation

Simply put, observing means collecting information about the environment. The result of the observation

⁴ This mapping is not unique, because of the arbitrary phase. All possible maps work equally well however, because the chosen phase is global and commutes with the evolution.

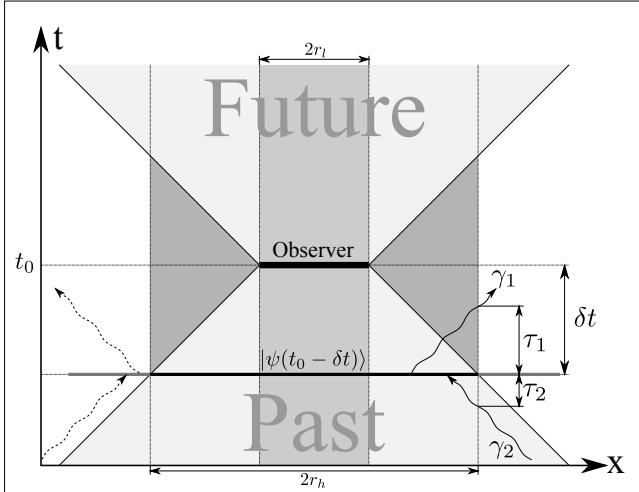


Figure 1: The observer reconstructs the state $|\psi\rangle$ based on the information collected from past interactions. The photons γ_1 and γ_2 transport information to and from the environment beyond the observation horizon.

is a time dependent description of the observed system state. This process is based on interactions and limited by their finite propagation speed in a relativistic universe. Hence the reconstructed state refers to a time slice in the past of the observer. The time asymmetry is created by the requirement of a memory to consolidate the state information. This arrow of time does not affect the measurement process at this point however and reconstructing a future state leads to the same fundamental properties.

Figure 1 shows the situation in spacetime. The observer with radius r_l reconstructs the state $|\psi(t_0 - \delta t)\rangle$ at the time t_0 from information in his past light cone. The observation delay δt and the radius of the reconstructed state horizon are related by $r_h = r_l + c\delta t$. During an observation the delay is held constant so that the observed system evolves on the same time base as the observer. Anything outside the world volume swept by the space inside the horizon is not part of the observer's subjective reality.

In a typical experimental setup the observer focuses on a more or less isolated object at relative rest. It is a reasonable approximation to assume that the only exchange of information with the unobserved universe is due to electromagnetic interactions. The two photons shown on the left are not having any effect on the reconstructed reality of the observer because their interaction is limited to the spatial region outside the reconstruction horizon. This is not true for the photons γ_1 and γ_2 . They interact with the reconstructed state and the corresponding duration of interaction with the world volume of the state are τ_1 and τ_2 . During that time

the state reconstruction performed by the observer will change in a non-unitary way. In addition, the observer will not be able to directly reconstruct the existence of either photon, because it does not pass through his own world volume.

As discussed before, the reconstructed state is described by a single Hilbert space vector. The non-unitary evolution will thus change the vector norm, just like we would expect from a measurement interaction like the one described by the measurement postulate. The escaping and incoming photons are also of unknown state, so that a random element is introduced into the reconstructed state. This is also in agreement with the measurement postulate. It seems natural to conjecture a relationship between the interaction with photons and what we call quantum measurement. The next chapter discusses the relevant processes in more detail.

IV. INTERACTIONS BETWEEN RADIATION AND MATTER

Let us consider a system consisting of only a single qubit. There is nothing else the observer can use to compare the qubit to, not even the observer itself exists as part of the quantum system in this simple model. The qubit is the entire observable universe, and hence we can apply the reconstruction procedure to it. Still, the qubit is not completely insulated as we assume it couples to the radiation field. The interaction is described by a unitary evolution on the entire unobserved universe. To keep it simple, the only states of the radiation field that are considered are inaccessible. The incoming state can be thought of as a single photon with a 2-dimensional representation of its polarization. The outgoing states can more complicated than that, depending on the scattering process. We call these scattering processes elementary, because only two qubits, one for the photon and one local to the observer, are given as the input state.

We will discuss three of these elementary scattering processes explicitly. In all of them the state of the local qubit is represented in the orthonormal basis $\{|0\rangle, |1\rangle\}$, which is also the physically preferred basis for the resulting process. The incoming photon state will be written as a linear combination of the orthonormal basis states $\{|\leftrightarrow\rangle, |\updownarrow\rangle\}$. For the outgoing radiation states an arbitrary orthonormal basis $\{|\rightsquigarrow_n\rangle : n \in \mathbb{N}\}$ is used. The vacuum state is $|o\rangle$. All processes are unitary because they map orthogonal states in the input space to orthogonal states in the output space. The input state corresponds to the objective state of the universe just before the interaction and the output state is the

objective state just after the interaction. Both input and output radiation states are locally inaccessible and will be removed in the local state reconstruction so that we can study the scattering process as observed locally. The state of the photon is considered to be entirely unknown.

The general input state in this setup is

$$|\psi\rangle := (\alpha |\uparrow\rangle + \beta |\leftrightarrow\rangle) (a |0\rangle + b |1\rangle) \quad (39)$$

$$= \alpha a |\uparrow\rangle |0\rangle + \alpha b |\uparrow\rangle |1\rangle + \beta a |\leftrightarrow\rangle |0\rangle + \beta b |\leftrightarrow\rangle |1\rangle \quad (40)$$

for $\alpha, \beta, a, b \in \mathbb{C}$ with:

$$\alpha\alpha^* + \beta\beta^* > 0 \quad (41)$$

$$aa^* + bb^* > 0 \quad (42)$$

The local reconstruction of $|\psi\rangle$ is then simply

$$\bar{\Lambda}(|\psi\rangle) = T((a |0\rangle + b |1\rangle) |\circ\rangle) \quad (43)$$

which is equivalent to the local qubit state, just as expected.

A. The uniform elementary scattering process

Consider the following mapping of the basis states from the input to the output space:

$$U_u : \begin{Bmatrix} |\uparrow\rangle |0\rangle \\ |\uparrow\rangle |1\rangle \\ |\leftrightarrow\rangle |0\rangle \\ |\leftrightarrow\rangle |1\rangle \end{Bmatrix} \mapsto \begin{Bmatrix} |0\rangle |\rightsquigarrow_1\rangle \\ |0\rangle |\rightsquigarrow_2\rangle \\ |1\rangle |\rightsquigarrow_3\rangle \\ |1\rangle |\rightsquigarrow_4\rangle \end{Bmatrix} \quad (44)$$

The objective input state $|\psi\rangle$ is then unitarily mapped to

$$U_u |\psi\rangle = \alpha a |0\rangle |\rightsquigarrow_1\rangle + \alpha b |0\rangle |\rightsquigarrow_2\rangle + \beta a |1\rangle |\rightsquigarrow_3\rangle + \beta b |1\rangle |\rightsquigarrow_4\rangle \quad (45)$$

Without taking the limit, the reconstructed local state of the output is

$$\Lambda(U_u |\psi\rangle) = T(\alpha a |0\rangle |\circ\rangle) + T(\alpha b |0\rangle |\circ\rangle) \quad (46)$$

$$+ T(\beta a |1\rangle |\circ\rangle) + T(\beta b |1\rangle |\circ\rangle) \\ = \alpha\alpha^* (aa^* + \beta\beta^*) |0\rangle \langle 0| |\circ\rangle \langle \circ| \\ + \beta\beta^* (aa^* + \beta\beta^*) |1\rangle \langle 1| |\circ\rangle \langle \circ| \quad (47)$$

Now taking the limit we get

$$\bar{\Lambda}(U_u |\psi\rangle) = \begin{cases} |0\rangle |\circ\rangle & \text{if } \alpha\alpha^* > \beta\beta^* \\ |1\rangle |\circ\rangle & \text{if } \alpha\alpha^* < \beta\beta^* \end{cases} \quad (48)$$

The state of the photon is unknown, and both cases must be equally likely. The case of exact equality is of zero measure and can be ignored in the statistical interpretation. Summarizing we have a process U that acts locally like

$$U_u : a |0\rangle + b |1\rangle \mapsto \begin{cases} |0\rangle & \text{with probability } p_0 = \frac{1}{2} \\ |1\rangle & \text{with probability } p_1 = \frac{1}{2} \end{cases} \quad (49)$$

Notably the result does not depend on the state of the input qubit. A local observer just sees a fully random outcome that can be predicted only in terms of probabilities.

B. The maximum elementary scattering process

A small change in the unitary scattering produces a very different outcome:

$$U_m : \begin{Bmatrix} |\uparrow\rangle |0\rangle \\ |\uparrow\rangle |1\rangle \\ |\leftrightarrow\rangle |0\rangle \\ |\leftrightarrow\rangle |1\rangle \end{Bmatrix} \mapsto \begin{Bmatrix} |0\rangle |\rightsquigarrow_1\rangle \\ |1\rangle |\rightsquigarrow_2\rangle \\ |0\rangle |\rightsquigarrow_3\rangle \\ |1\rangle |\rightsquigarrow_4\rangle \end{Bmatrix} \quad (50)$$

The objective input state $|\psi\rangle$ is mapped to

$$U_m |\psi\rangle = \alpha a |0\rangle |\rightsquigarrow_1\rangle + \alpha b |1\rangle |\rightsquigarrow_2\rangle + \beta a |0\rangle |\rightsquigarrow_3\rangle + \beta b |1\rangle |\rightsquigarrow_4\rangle \quad (51)$$

which is stripped to result in

$$\Lambda(U_m |\psi\rangle) = T(\alpha a |0\rangle |\circ\rangle) + T(\alpha b |1\rangle |\circ\rangle) \quad (52)$$

$$+ T(\beta a |0\rangle |\circ\rangle) + T(\beta b |1\rangle |\circ\rangle) \\ = aa^* (\alpha\alpha^* + \beta\beta^*) |0\rangle \langle 0| |\circ\rangle \langle \circ| \\ + bb^* (\alpha\alpha^* + \beta\beta^*) |1\rangle \langle 1| |\circ\rangle \langle \circ| \quad (53)$$

and eventually results in the limit

$$\bar{\Lambda}(U_m |\psi\rangle) = \begin{cases} |0\rangle |\circ\rangle & \text{for } aa^* > bb^* \\ |1\rangle |\circ\rangle & \text{for } aa^* < bb^* \end{cases} \quad (54)$$

so that we get the local observation

$$U_m : a |0\rangle + b |1\rangle \mapsto \begin{cases} |0\rangle & \text{if } |a| > |b| \\ |1\rangle & \text{if } |b| > |a| \end{cases} \quad (55)$$

The unknown photon state does not influence the outcome and the process is fully deterministic, even for a local observer. The result is a projection on the dominant component of the qubit in the preferred basis.

C. The Born elementary scattering process

The two scattering processes discussed above are only using information from either the qubit or the photon state to produce the output. In this section we will discuss a process that mixes the influence of both equally and results in a statistical rule that also depends on the state of the qubit. The map that creates this behavior is

$$U_B : \begin{Bmatrix} |\uparrow\rangle |0\rangle \\ |\uparrow\rangle |1\rangle \\ |\leftrightarrow\rangle |0\rangle \\ |\leftrightarrow\rangle |1\rangle \end{Bmatrix} \mapsto \begin{Bmatrix} |0\rangle |\rightsquigarrow_1\rangle \\ (|1\rangle |\rightsquigarrow_2\rangle + |0\rangle |\rightsquigarrow_3\rangle) / \sqrt{2} \\ (|0\rangle |\rightsquigarrow_4\rangle + |1\rangle |\rightsquigarrow_5\rangle) / \sqrt{2} \\ |1\rangle |\rightsquigarrow_6\rangle \end{Bmatrix} \quad (56)$$

and produces the output state

$$U_B |\psi\rangle = \alpha a |0\rangle |\rightsquigarrow_1\rangle + \beta b |1\rangle |\rightsquigarrow_6\rangle \\ + \alpha b (|1\rangle |\rightsquigarrow_2\rangle + |0\rangle |\rightsquigarrow_3\rangle) / \sqrt{2} \\ + \beta a (|0\rangle |\rightsquigarrow_4\rangle + |1\rangle |\rightsquigarrow_5\rangle) / \sqrt{2} \quad (57)$$

which strips to

$$\Lambda(U_B |\psi\rangle) = T(\alpha a |0\rangle |\circ\rangle) \\ + \frac{1}{2} (T(\alpha b |1\rangle |\circ\rangle) + T(\alpha b |0\rangle |\circ\rangle)) \\ + \frac{1}{2} (T(\beta a |0\rangle |\circ\rangle) + T(\beta a |1\rangle |\circ\rangle)) \\ + T(\beta b |1\rangle |\circ\rangle) \\ = \left(\alpha \alpha^* a a^* + \frac{1}{2} (\alpha \alpha^* b b^* + \beta \beta^* a a^*) \right) |0\rangle \langle 0| |\circ\rangle \langle \circ| \\ + \left(\beta \beta^* b b^* + \frac{1}{2} (\alpha \alpha^* b b^* + \beta \beta^* a a^*) \right) |1\rangle \langle 1| |\circ\rangle \langle \circ| \quad (58)$$

and after taking the limit results in

$$\bar{\Lambda}(U_B |\psi\rangle) = \begin{cases} |0\rangle |\circ\rangle & \text{if } \alpha \alpha^* a a^* > \beta \beta^* b b^* \\ |1\rangle |\circ\rangle & \text{if } \alpha \alpha^* a a^* < \beta \beta^* b b^* \end{cases} \quad (59)$$

The outcome of the scattering process depends on both the amplitudes of the qubit in the preferred basis and the unknown incoming photon state.

We do not have any information about the state of the photon. That implies that the statistical distribution of α and β does not depend on the choice of a basis, or in other words, the distribution of (α, β) must be invariant under $SU(2)$ transforms.

One possible distribution⁵ that realizes this symmetry

is given by

$$\alpha := G_1 + iG_2 \quad (60)$$

$$\beta := G_3 + iG_4 \quad (61)$$

Where G_n are mutually independent identically distributed gaussian random variables with zero mean. Because the sum of two independent Gaussian variables results in a new Gaussian variable, this construction is invariant under unitary transformations. The case $\alpha = \beta = 0$ leads to an invalid state, but is of zero measure and we can safely ignore it. The magnitude of a complex gaussian random variable is Rayleigh distributed, so that we have

$$|\alpha| = R_1 \quad (62)$$

$$|\beta| = R_2 \quad (63)$$

with two equally distributed independent Rayleigh variables R_1 and R_2 . The probability density function of each is

$$f(x) = x \exp\left(-\frac{x^2}{2}\right) \quad (64)$$

The probability $p(|a| |\alpha| > |b| |\beta|)$ can then be expressed in terms of the probability density functions:

$$p(|a| |\alpha| > |b| |\beta|) = \int_0^\infty \int_0^{\frac{|a|}{|b|} x_1} f(x_1) f(x_2) dx_2 dx_1 \quad (65)$$

$$= \frac{|a|^2}{|a|^2 + |b|^2} \quad (66)$$

and of course for the complementary event

$$p(|a| |\alpha| < |b| |\beta|) = \frac{|b|^2}{|a|^2 + |b|^2} \quad (67)$$

The local observation is therefore

$$U_B : a |0\rangle + b |1\rangle \mapsto \begin{cases} |0\rangle & \text{with } p_0 = \frac{|a|^2}{|a|^2 + |b|^2} \\ |1\rangle & \text{with } p_1 = \frac{|b|^2}{|a|^2 + |b|^2} \end{cases} \quad (68)$$

which constitutes the Born rule.

V. LOCAL QUANTUM THEORY AND REALITY

A. Emergent reality and disrupted time

In the previous section we have been able to demonstrate the emergence of the Born rule from the local reconstruction of an evolving universe's state. The analysis of the scattering processes has been restricted to

⁵ We do not require a normalized photon state. So the distribution of the magnitudes does not make a difference as long as the $SU(2)$ symmetry is realized.

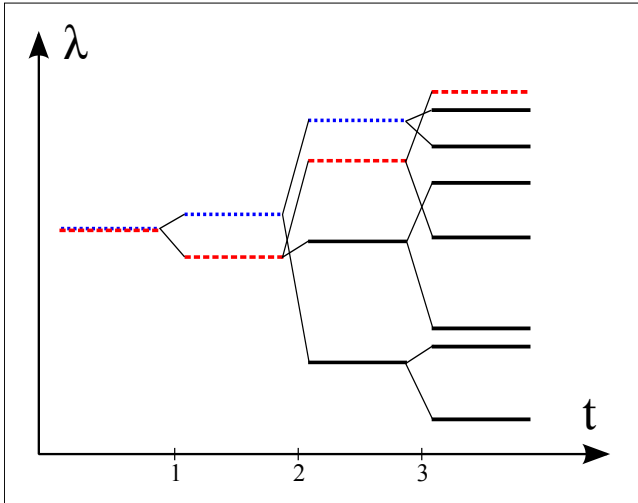


Figure 2: Evolution of the branches and their eigenvalues under Born scattering. The scattering events occur at times 1, 2 and 3 and they result in new branches containing the records of the previous history the respective branch. The dotted blue lines show the dominant branch and its history after the first and second scattering events. One event later the red dashed line becomes the dominant branch with its own history. The single initial branch is part of both histories.

a single qubit in an otherwise unpopulated observer regime and an initially pure quantum state. After one scattering pass the state will not be objectively pure anymore but instead consist of two branches encoded by the eigenvectors of the stripped state. A second pass will again map to the only two possible local output branches and the eigenvalues will mix between the two previous branches. We cannot expect the Born rule to hold under these conditions.

A minor modification to the local system takes care of this problem and also makes our assumptions more realistic. An observer that wants to test for validity of the Born rule has to be able to compare sequential observation results, meaning he has to have access to a recorded history of events. Adding a memory to the local state of the observer makes the different outcomes of branches belonging to different iterations distinguishable and keeps the branches from mixing. Hence the number of branches doubles after each *recorded* scattering process.

Figure 2 shows a sequence of Born scattering processes and the resulting change of the reconstructable branch the local observer is aware of. Looking only at the sequence of dominant branches one can verify that the Born rule holds for the transitions 1 and 2 because the new dominant branch is created from the previous dominant branch, as the situation is identical to the elementary scattering event with a single branch initial state.

That does not hold true for the third scattering process. The formerly suppressed branch splits into the new dominant branch. Depending on the actual states this sequence of branches usually *breaks* the Born rule.

Despite the partly broken Born rule for the sequence of reconstructed states the observer will never see any results that disagrees with the Born rule. The reason for this lies in the way the observer tests the statistics of the results. He keeps a list of old results to compare to the new ones, and the branch switch not only determines the current observed result but also the list of remembered states. This is also illustrated in figure 2. After the third branch switch the observer remembers the history marked with the dashed red line, and this history describes a sequence of observation outcomes that is in agreement with the Born rule.

Everything that the observer would consider as part of his reality is contained in the current reconstructed state of the universe and his memory. We can therefore call the sequence of branches that lead to the current reconstructed branch and that is stored in the memory of the observer an emergent subjective reality. A branch switch will not only change the current perception but also the perception of history leading up to this state. The observer can switch between realities without even noticing, because all records will agree with the newly formed reality. This picture leaves subjective time and history disrupted by observed scattering events. This is a drastic but seemingly unavoidable consequence of observation. A rough estimation of probabilities however suggests that an event uncovering a long time hidden reality branch is very unlikely. So the observer's subjective history is stable in the long run.

B. Macroscopic interactions and quantum measurement

The three elementary scattering processes with locally non-unitary outcome described here all acted on single qubit systems. While their mechanisms are very similar, Born scattering comes with the unique property of scalability.

Consider a set of projectors

$$\mathbb{P} = \{P_n : n \in \mathbb{N}\} \quad (69)$$

acting on the Hilbert space \mathcal{H}_P with the constraint that:

$$[P_m, P_n] = 0 \quad (70)$$

We also define the identity operator \mathbb{I}_P on \mathcal{H}_P and the complementary projectors

$$\bar{P}_n = \mathbb{I}_P - P_n \quad (71)$$

We call this set of projectors *complete* if there is a special orthonormal basis $\{|k_P\rangle\}$ of \mathcal{H}_P and we can find two (disjoint) sets $I_1, I_2 \subset \mathbb{N}$ so that

$$|k_P\rangle \langle k_P| = \prod_{n \in I_1} P_n \prod_{m \in I_2} \bar{P}_m \quad (72)$$

for all k . The set is *independent* if we cannot remove any projectors from \mathbb{P} without giving up completeness.

With the set of projectors there is a natural way to define a unitary evolution to split up vectors in \mathcal{H}_P .

$$U_n : |\psi\rangle \mapsto P_n |\psi\rangle \otimes |0\rangle + \bar{P}_n |\psi\rangle \otimes |1\rangle \quad (73)$$

Here $\{|0\rangle, |1\rangle\}$ is the basis of a qubit⁶ and we can apply the Born scattering evolution to it, with the local result

$$\bar{\Lambda}(U_B U_n |\psi\rangle) = \begin{cases} P_n |\psi\rangle |0\rangle & \text{with } p = \frac{\langle \psi | P_n | \psi \rangle}{\langle \psi | \psi \rangle} \\ \bar{P}_n |\psi\rangle |1\rangle & \text{with } p = \frac{\langle \psi | \bar{P}_n | \psi \rangle}{\langle \psi | \psi \rangle} \end{cases} \quad (74)$$

We have seen that the recorded subjective observation of qubits is consistent and stable, allowing us to restrict our discussion to the dominant branch and taking the position of the local observer. Repeating the observation with a different projector P_m and a fresh qubit maps the first branch to $P_m P_n |\psi\rangle |0\rangle |0\rangle$ with the old qubit state as the last factor. For a cleaner notation we write the qubit state ordered list inside a single ket $|0, 0\rangle$. The probability of finding this branch combines the probabilities from both scattering events and results in

$$p = \frac{\langle \psi | P_n^\dagger P_m^\dagger P_m P_n | \psi \rangle}{\langle \psi | P_n | \psi \rangle} \cdot \frac{\langle \psi | P_n | \psi \rangle}{\langle \psi | \psi \rangle} \quad (75)$$

$$= \frac{\langle \psi | P_m P_n | \psi \rangle}{\langle \psi | \psi \rangle} \quad (76)$$

As can be seen from generalizing this calculation, further scatterings only result in adding further projectors to the numerator of the probability expression. The order is arbitrary because the projectors commute by definition. This is also true for the projectors in front of the state $|\psi\rangle$. We can therefore choose the canonical ordering of the index without changing the properties of the result as far as they relate to the state $|\psi\rangle$. The order of the qubit history will change however. This motivates the definition of the operator

$$M = \prod_{n=1}^N U_B U_n \quad (77)$$

⁶ When these evolutions are concatenated, the qubit must be assumed to be a different one in each stage. This is not reflected in our notation in order to keep it simple.

where each factor comes with a fresh qubit. We also define $|[j]\rangle$ as the qubit list with the N -digit binary expansion of j . Similarly we define $P_{[j]}$ to be the product sequence with the digits $\{P_n, \bar{P}_n\}$ following the N -digit binary representation of j .

The subjective local result of the application of M on the state $|\psi\rangle$ is then

$$\bar{\Lambda}(M |\psi\rangle) = \begin{cases} P_{[0]} |\psi\rangle |[0]\rangle & \text{with } p = \frac{\langle \psi | P_{[0]} | \psi \rangle}{\langle \psi | \psi \rangle} \\ P_{[1]} |\psi\rangle |[1]\rangle & \text{with } p = \frac{\langle \psi | P_{[1]} | \psi \rangle}{\langle \psi | \psi \rangle} \\ \vdots & \vdots \\ P_{[2^N-2]} |\psi\rangle |[2^N-2]\rangle & \text{with } p = \frac{\langle \psi | P_{[2^N-2]} | \psi \rangle}{\langle \psi | \psi \rangle} \\ P_{[2^N-1]} |\psi\rangle |[2^N-1]\rangle & \text{with } p = \frac{\langle \psi | P_{[2^N-1]} | \psi \rangle}{\langle \psi | \psi \rangle} \end{cases} \quad (78)$$

We are interested in the limit of very large N . Then the $P_{[k]}$ contain all possible projector lists as sublists. For a given list of projections that multiplies to a projector on a single dimensional subspace we can find a *unique* binary sequence that produces this list as a sublist and preserves that subspace in the remaining projectors, because either P or \bar{P} preserves the subspace. All other lists containing the same sublist must multiply to 0.

The consequence is that if we have a *complete* set of projectors \mathbb{P} then there is exactly one outcome with non-zero probability resulting in the state $|k_P\rangle \otimes |[j]\rangle$ with a probability of

$$p = \frac{\langle \psi | k_P \rangle \langle k_P | \psi \rangle}{\langle \psi | \psi \rangle} \quad (79)$$

Summarized, for *complete* \mathbb{P} and sufficiently large N the scattering iteration

$$\bar{\Lambda}(M |\psi\rangle) = |k_P\rangle \otimes |[j]\rangle \quad \text{with } p = \frac{|\langle k_P | \psi \rangle|^2}{\langle \psi | \psi \rangle} \quad (80)$$

results in the *Born rule* for measurement in the basis $\{|k_P\rangle\}$.

We have constructed a measurement mechanism that works for Hilbert spaces of arbitrary size and can therefore be applied to macroscopic systems. The mechanism does not need very special initial condition or careful tuning. In fact, it is very robust, as the order of the elementary scattering processes does not change the outcome nor does the actual choice of projectors. A physical device realizing this mechanism should not be hard to design and build.

We only discussed the case of a *complete* set of projectors. The framework presented allows for generalizations that are not be discussed here however. It is interesting to note that the Born rule is the only rule that delivers robust and consistent results for a macroscopic system built from single qubit interactions, mostly due to the canceling terms in equation (75).

VI. DISCUSSION

We have seen that it is possible to derive the measurement postulate of quantum theory by assuming the perspective of an observer inside the system. It has also been shown that the indeterminism we observe is only subjective. The global theory can be fully deterministic and yet we are only able to see a fundamentally random universe. What is more, the Born rule arises naturally as the only sustainable statistical law for macroscopic systems and its physical mechanism is so general and robust that it can be expected to be found in many real world systems. Throughout the derivation no additional unmotivated assumptions have been made, so that we can regard the results as a general solution to the measurement problem.

The interactions responsible for the measurement process are well localized scattering events. This explains the observed dynamic position superselection rule that has not been understood well before. We also showed that the position basis is special for reducing the quantum state as dictated by the light cone structure of spacetime. The resulting law for stripping the state generalizes the Von Neumann construction [6] of a local state by tracing over tensor factor spaces containing inaccessible information.

The results go beyond the measurement postulate too. Other non-unitary laws for *two-level systems* have been derived. While only two have been discussed here, there are many hybrid forms that can also be realized. The laws discussed here could be very helpful for analyzing the dynamics of atomic spectroscopy. It seems possible that they are more appropriate than the Born process for describing the quantum jumps observed there.

The arrow of time of the measurement process has been inherited from the arrow of time of radiation, and we have seen that the arrow of memory records has to be identical to the measurement arrow. Only in this manner can a subjective reality according to the Born rule appear. The observed reality is found to be subjective and discontinuous, even allowing for jumps between histories while the long term history is unique and stable. Even though similarities to Everett's relative state interpretation of quantum theory [2] are obvious, the theory laid out here only has one observer and one reality at each time. The branches of reality discussed here are similar to many worlds, but are hidden so that they are not realized for the observer, and also not for any other observer residing nearby. We also found that decoherence does not play a role in the measurement process

and might not even be required for understanding the macroscopic properties of a quantum system.

Besides being a very simple theory and therefore preferred by Occam's razor, the theory presented here is testable. We make enough additional predictions about the subjective description of elementary quantum processes and most importantly about the construction of a Born rule compliant measurement device, so it should be possible to design experiments for verification.

Lastly, the work presented here only shows a small part of the possible applications of the general idea. We have not discussed the implications of measurement on distant entangled states, the influence of information sinks like black holes or the possible role of the neutrino as a carrier of information into the environment, the possibilities for better experimental measurement methods or a better understanding of atomic spectroscopy. There are many open questions, but the foundations have been laid.

Acknowledgments

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